Mathematics 552

Quiz 2 **Name:**________________________________

You must show your work to get full credit.

1. Let h be a differentiable function of x and y defined on an open set U. Give the limit definition of the following:

(a) $\frac{\partial h}{\partial x}(x, y) =$

(b) $\frac{\partial h}{\partial y}(x, y) =$

2. Let $f(z)$ be a complex valued function defined on a open set U of $\mathbb{C}$. Give the limit definition of the ***complex derivative*** $f'(z_0)$.

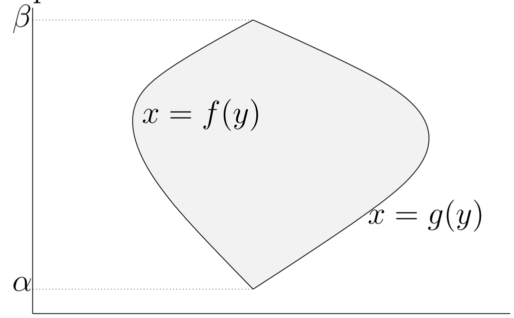
$f'(z_0) =.$

3. Let $f(z) = u + iv$ be defined on the open set U of $\mathbb{C}$.

(a) Define what it means for the ***Cauchy-Riemann*** equations to hold at $z \in U$.

(b) Prove if $f(z)$ is complex differentiable at z_0, then the Cauchy-Riemann equations hold at z_0.

4. Let $Q(x,y)$ have continuous partial derivatives and let D be the domain below:



Prove $\int_{\partial D} Q(x,y)\,dy = \iint_D Q_x(x,y)\,dx\,dy$.

5. Use Green's formula

$$\int_{\partial D} P\,dx + Q\,dy = \iint_D \left(-P_y + Q_x\right)\,dx\,dy$$

to show that if a function $f = u + iv$ satisfies the Cauchy-Riemann equation on a bounded open set U with nice boundary that

$$\int_{\partial D} f(z)\,dz = 0.$$