Mathematics 172 Homework

Here we will mostly review some facts about exponential functions. Recall that these are functions of the form:

$$P_t = P_0\lambda^t.$$

This is implies

$$P_{t_1} = P_0\lambda^{t+1} = \lambda P_0\lambda^t = P_t.$$

Conversely we have seen in class that if $P_{t+1} = \lambda P_t$, then for positive integers $P_t = P_0\lambda^t$. Thus

$$P_t \text{ is an exponential on integers if and only if } \frac{P_{t+1}}{P_t} \text{ is constant.}$$

This gives us a easy way to check if a function is an exponential. For example consider the function given by the table

t	0	1	2	3	4
P_t	25.00	30.00	36.00	43.20	51.84

We compute the ratios $\frac{P_{t+1}}{P_t}$ to three sufficient digits for the values of t we have to get

$$\frac{P_1}{P_0} = 1.20, \quad \frac{P_2}{P_1} = 1.20, \quad \frac{P_3}{P_2} = 1.20 \quad \frac{P_4}{P_3} = 1.20$$

and so this ratio is constant. This shows that at least for the values of t given that P_t is the function

$$P_t = 25(1.2)^t.$$

For anther example consider the table

t	0	1	2	3	4
P_t	5.20	5.04	4.89	4.75	4.60

This time the ratios are

$$\frac{P_1}{P_0} = 1.20, \quad \frac{P_2}{P_1} = 1.20, \quad \frac{P_3}{P_2} = 1.20 \quad \frac{P_4}{P_3} = 1.20$$

We again compute the ratios to three sufficient digits.

$$\frac{P_1}{P_0} = .968, \quad \frac{P_2}{P_1} = .970, \quad \frac{P_3}{P_2} = .969, \quad \frac{P_4}{P_3} = .970$$

which is close enough to being constant. This function is basically the same as

$$P_t = 5.2(.97)^t.$$

1. Is the following an exponential function? Explain why.

t	0	1	2	3	4
N_t	47.9	58.9	72.46	89.1	109.6

2. Is the following an exponential function? Explain why.

t	0	1	2	3	4
Q_t	32.1	35.2	37.8	39.3	43.7

3. Is the following an exponential function? Explain why.

t	0	1	2	3	4
P_t	43.00	37.41	32.45	28.31	24.63